Externalities

Extendities and the Market

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Presentation Outline

- **1** Markets Mathematically
 - 2 Externalities
- **3** Public Goods
- **4** Conclusion

Market Supply (Mathematical)

General Form:

$$Q_S = g(P, C, T, E, Z)$$

where:

- Q_S: Quantity supplied.
- *P*: Price of the good (own price).
- C: Cost of production inputs.
- T: Technology level.
- E: Expectations about future prices.
- Z: Number of producers or external factors (e.g., policy, weather).

Price Elasticity of Supply:

$$\varepsilon_{S} = \frac{\partial Q_{S}}{\partial P} \cdot \frac{P}{Q_{S}}$$

Measures responsiveness of supply to price changes.

Market Demand (Mathematical)

General Form:

$$Q_D = f(P, Y, P_s, P_c, T, E, N)$$

where:

- Q_D: Quantity demanded.
- P: Price of the good (own price).
- Y: Consumer income.
- P_s: Prices of substitutes.
- P_c : Prices of complements.
- T: Consumer tastes and preferences.
- E: Expectations about future prices/income.
- N: Number of buyers.

Market Demand (Mathematical)

Price Elasticity of Demand:

$$\varepsilon_D = \frac{\partial Q_D}{\partial P} \cdot \frac{P}{Q_D}$$

- Measures responsiveness of demand to price changes.
- $|\varepsilon_D| > 1$: Elastic demand.
- $|\varepsilon_D| < 1$: Inelastic demand.

Linear Demand Example:

$$Q_D = a - bP, \quad b > 0$$

- a: Intercept, represents maximum demand when P = 0.
- b: Slope, reflects sensitivity to price changes.

Market Equilibrium Graphically



Figure 1: Supply and demand!

Market Equilibrium: Example Problem

Problem Setup:

- **Demand Equation:** $Q_D = 50 2P$
- Supply Equation: $Q_S = 10 + 3P$
- Find the equilibrium price (P^*) and quantity (Q^*) .

Solution:

• At equilibrium, $Q_D = Q_S$:

$$50 - 2P = 10 + 3P$$

Simplify and solve for P*:

 $50-10=3P+2P \Rightarrow 40=5P$ $\Rightarrow P^* = 8$

• Substitute P^{*} into either equation to find Q^{*}: • Using Q_D :

$$Q^* = 50 - 2(8) = 34$$

Producer Surplus and Consumer Surplus

• **Consumer Surplus** is the difference between what consumers are willing to pay for a good or service and what they actually pay. It represents the benefit to consumers from participating in the market.

$$CS = \int_0^Q P(Q) \, dQ - P_{market} imes Q$$

where P(Q) is the price consumers are willing to pay at each quantity.

• **Producer Surplus** is the difference between the price producers receive for a good or service and the minimum price at which they are willing to sell it. It represents the benefit to producers from participating in the market.

$$PS = P_{market} \times Q - \int_0^Q C(Q) \, dQ$$

where C(Q) is the cost to producers at each quantity.

Worked Example: Consumer Surplus

Consider a market for coffee, where the demand curve is given by:

P(Q) = 100 - 2Q

and the market price is P = 40. We need to find the consumer surplus. 1. First, find the equilibrium quantity where the demand price equals the market price. Set:

$$100 - 2Q = 40 \quad \Rightarrow \quad Q = 30$$

2. The consumer surplus is the area of the triangle under the demand curve, from Q = 0 to Q = 30, and above the price of 40. The formula for consumer surplus is:

$$CS = \frac{1}{2} \times \mathsf{Base} \times \mathsf{Height} = \frac{1}{2} \times 30 \times (100 - 40) = 900$$

Therefore, the consumer surplus is 900.

Extenalities

Worked Example: Producer Surplus

Consider the same market for coffee. The supply curve is given by:

P(Q) = 10 + Q

and the market price is P = 40. We need to find the producer surplus. 1. First, find the equilibrium quantity where the supply price equals the market price. Set:

$$10 + Q = 40 \quad \Rightarrow \quad Q = 30$$

2. The producer surplus is the area of the triangle under the market price, from Q = 0 to Q = 30, and above the supply curve. The formula for producer surplus is:

$$PS = P_{market} imes Q - \int_0^Q (10+Q) \, dQ$$

Solving for Producer Surplus

3. Compute the integral:

$$PS = 40 \times 30 - \left[10Q + \frac{Q^2}{2}\right]_0^{30}$$
$$PS = 1200 - \left(10 \times 30 + \frac{30^2}{2}\right) = 1200 - (300 + 450) = 1200 - 750 = 450$$

Therefore, the producer surplus is 450.

Overview of Coase's Theorem

- Proposed by economist Ronald Coase in 1960.
- States that under certain conditions, private bargaining will result in an efficient allocation of resources, regardless of initial property rights.
- The theorem suggests that externalities can be resolved by private parties if transaction costs are low and property rights are well-defined.
- Emphasizes the role of property rights and transaction costs in solving environmental and resource allocation problems.

Mathematical Explanation of Coase's Theorem

- Let X represent the level of an externality (e.g., pollution) and C(X) be the cost to the affected party.
- The benefit to the polluter can be represented as B(X).
- An efficient outcome occurs when marginal benefit (MB) equals marginal cost (MC), i.e., B'(X) = C'(X).
- If transaction costs are zero, the parties can negotiate to maximize joint surplus:

$$\max_X B(X) - C(X)$$

• The final level of X is independent of the initial allocation of rights but depends on achieving B'(X) = C'(X).

Markets Mathematically

Intuition Behind the Proof of Coase's Theorem

- The core idea is that if property rights are clearly defined, individuals can negotiate to reach a mutually beneficial solution.
- Consider two parties: a polluter and a harmed party. Without any intervention, the polluter has no incentive to reduce pollution.
- If property rights are assigned (to either party), the two can bargain. They will exchange payments to reduce pollution if the cost to the polluter is less than the harm avoided.
- Through bargaining, resources (pollution control) are reallocated until no further gains can be made (efficiency).
- The initial distribution of property rights only affects the distribution of wealth, not the level of pollution in the final outcome.
- Thus, the theorem highlights that externalities can be internally corrected by the parties involved, leading to an efficient outcome.

Assumptions Behind Coase's Theorem

- Low Transaction Costs: Bargaining and enforcement costs are negligible, allowing parties to negotiate freely.
- Clearly Defined Property Rights: Property rights must be assigned and enforceable for negotiation to occur.
- Rational Behavior: Parties are assumed to act rationally to maximize their individual benefits.
- No Wealth Effects: Initial distribution of property rights does not affect parties' preferences over outcomes.
- **Perfect Information:** All parties know the costs and benefits of externalities to all affected individuals.

When Coase's Theorem Fails

- Not all market participants can participate in markets. Our children, animals, the environment.
- Not everything can be divided. Stiglitz Nobel prize.
- Very difficult to create some markets and to evaluate fairness

What is a Market Failure?

- Normally in economics, we assume that markets create optimal outcomes for society, when this does not happen, the market has failed.
- Market failures occur when the assumptions of perfect competition are violated or markets are missing
- Classic examples: Monopoly (particularly natural monopoly), public goods, externalities, incomplete information (moral hazard and adverse selection)

What is a Market Failure?

- Normally in economics, we assume that markets create optimal outcomes for society, when this does not happen, the market has failed.
- Market failures occur when the assumptions of perfect competition are violated or markets are missing
- Classic examples: Monopoly (particularly natural monopoly), public goods, externalities, incomplete information (moral hazard and adverse selection)
- Traditionally, market failure is judged by Pareto efficiency. Is this right as a framework?

How Policy Fixes Market Failures

- Economists are not always anti-government intervention
- Economists generally acknowledge that governments have a role in fixing market failures
- Mandating information in car market, taxes in subsidies with externalities, stopping collusion

How Policy can Create Market Failure

- Policy can distort market incentives and have unintended consequences
- Sometimes taxes, market regulation, and regulatory capture
- Example: Catalytic Converter
- Policy can and is effective but important to think critically about effects before implementation

What is an Externality?

- Externalities are costs or benefits incurred by someone who did not participate in a transaction
- Externalities are a major impediment to market efficiency and provide an example of when regulation works well

Example of Negative externality

- Pollution
- Smoking
- Traffic Congestion



Figure 2: Restaurants are better without smoking sections

Public Goods

Conclusion

Example of Positve Externality

- Smoke detectors
- Vaccines
- Education



Figure 3: Why should I care that my apartment is on fire?

Public Goods

Conclusion

Negative Externality Graphed



Figure 4: Negative Externaltiy

Positive Externality Graphed



Figure 5: Positive Externality

Positive Externality - Mathematical Formulation

A positive externality occurs when the consumption or production of a good or service causes a benefit to a third party.

The general form for the market outcome with a positive externality can be written as:

MSC = MPC - External Benefit

Where:

- *MSC* is the marginal social cost.
- MPC is the marginal private cost.
- The "External Benefit" represents the benefit to society, such as positive spillovers from education or innovation.

The socially optimal level of production/consumption is where:

MSC = MB

where MB is the marginal benefit to society.

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Negative Externality - Mathematical Formulation

A negative externality occurs when the consumption or production of a good or service imposes a cost on a third party.

The general form for the market outcome with a negative externality can be written as:

MPC = MSC + External Cost

Where:

- *MSC* is the marginal social cost.
- MPC is the marginal private cost.
- The "External Cost" represents the cost to society, such as pollution or resource depletion.

The socially optimal level of production/consumption is where:

$$MSC = MB$$

Worked Example: Positive Externality

Consider a scenario where education is a good with a positive externality. The marginal private benefit (MPB) of education is given by:

MPB = 100 + 2Q

The marginal social benefit (MSB) is higher due to the positive externality and is given by:

MSB = 100 + 3Q

The marginal cost (MC) of education is constant:

MC = 20

Socially Optimal Quantity

To find the socially optimal quantity, set MSB = MC:

$$100 + 3Q = 20$$

Therefore, the socially optimal quantity of education is 20 units, which accounts for the external benefit to society.

Worked Example: Negative Externality

Consider a scenario where a factory produces a good that creates pollution as a negative externality. The marginal private cost (MPC) of production is:

$$MPC = 10 + 2Q$$

The marginal social cost (MSC) includes the external cost of pollution, which is represented by:

MSC = 10 + 2Q + 5Q

The marginal benefit (MB) of production is:

MB = 50 - Q

Socially Optimal Quantity

To find the socially optimal quantity, set MSC = MB:

$$10 + 2Q + 5Q = 50 - Q$$
$$8Q = 40$$
$$Q = 5$$

Therefore, the socially optimal quantity of production is 5 units, which accounts for the external cost imposed on society.

Deadweight Loss

- Deadweight Loss is a loss to society without a corresponding gain
- Deadweight loss represents a policy failure
- You can improve social welfare just by fixing it
- You can make the world better just by fixing it
- Created when economic behavior is changed from socially optimal

Deadweight Loss Graphically



Figure 6: Deadweight Loss

Deadweight Loss - Concept

Deadweight loss represents the lost total surplus (consumer surplus + producer surplus) due to market inefficiency, often caused by price controls (such as price ceilings or price floors), taxes, or monopolies. In a competitive market, the equilibrium quantity and price maximize the total surplus. Deadweight loss occurs when the market is not operating at this efficient equilibrium due to external factors.

The general formula for deadweight loss is:

$$\mathsf{DWL} = rac{1}{2} imes (\mathsf{Price Change}) imes (\mathsf{Quantity Change})$$

Where:

- Price Change is the difference between the price consumers are willing to pay and the price they actually pay due to the market distortion.
- Quantity Change is the difference between the quantity produced or consumed with the distortion and the equilibrium quantity without the distortion.

Worked Example: Deadweight Loss with a Price Ceiling

Consider a market for apples. The demand curve is given by:

P(Q) = 100 - 2Q

and the supply curve is given by:

$$P(Q) = 20 + Q$$

The market equilibrium price and quantity are found by setting the demand and supply curves equal:

$$100 - 2Q = 20 + Q$$

Deadweight Loss with price Ceiling

Solving for Q, we get:

$$3Q = 80 \Rightarrow Q = 26.67$$

$$P = 100 - 2 \times 26.67 = 46.67$$

Now, suppose the government imposes a price ceiling of 40, *whichisbelowtheequilibriumprice*.

At the price ceiling of \$40, the quantity demanded is:

$$100 - 2 \times 40 = 20$$

The quantity supplied at this price is:

$$40 - 20 = 20$$
Deadweight Loss with price Ceiling

The quantity traded in the market is 20, which is less than the equilibrium quantity of 26.67. The deadweight loss is the area of the triangle formed by the equilibrium quantity and the new quantity supplied, with the price ceiling causing a price distortion. The formula for deadweight loss is:

$$DWL = \frac{1}{2} \times (Price Change) \times (Quantity Change)$$

Here, the price change is 46.67 - 40 = 6.67 and the quantity change is 26.67 - 20 = 6.67.

$$DWL = \frac{1}{2} \times 6.67 \times 6.67 = 22.22$$

Therefore, the deadweight loss is 22.22.

Worked Example: Deadweight Loss with a Tax

Consider the same market for apples, with a tax imposed on producers. The tax shifts the supply curve upward by the amount of the tax. Suppose the tax is \$10 per unit.

The new supply curve becomes:

Markets Mathematically

$$P(Q) = 20 + Q + 10 = 30 + Q$$

Now, we find the new equilibrium by setting the new supply curve equal to the demand curve:

$$100 - 2Q = 30 + Q$$

Worked Example: Deadweight Loss with a Tax

Solving for Q, we get:

 $3Q = 70 \Rightarrow Q = 23.33$

 $P = 100 - 2 \times 23.33 = 53.34$

With the tax, the price paid by consumers is 53.34, and the price received by producers is 43.34 (after the tax of 10).

The deadweight loss is the area of the triangle formed by the equilibrium quantity and the new quantity traded, with the tax causing a price distortion. The price change is 53.34 - 43.34 = 10 and the quantity change is 26.67 - 23.33 = 3.34.

Worked Example: Deadweight Loss with a Tax

The formula for deadweight loss is:

$$DWL = \frac{1}{2} \times (Price Change) \times (Quantity Change)$$

 $DWL = \frac{1}{2} \times 10 \times 3.34 = 16.7$

Therefore, the deadweight loss due to the tax is \$16.7.

Worked Example: Deadweight Loss with an Externality

Consider a market for manufacturing where the supply curve is given by:

P(Q) = 10 + 2Q

and the demand curve is given by:

$$P(Q) = 50 - Q$$

Without any externalities, the equilibrium quantity is found by setting the supply and demand curves equal:

$$50 - Q = 10 + 2Q$$

Worked Example: Deadweight Loss with an Externality

Solving for Q, we get:

 $3Q = 40 \Rightarrow Q = 13.33$ P = 50 - 13.33 = 36.67

Now, suppose the production of this good generates a negative externality that imposes a \$10 cost on society per unit produced. The marginal social cost (MSC) curve shifts up by the amount of the externality, so:

$$MSC = 10 + 2Q + 10 = 20 + 2Q$$

Worked Example: Deadweight Loss with an Externality

To find the socially optimal quantity, we set the marginal social cost equal to the demand curve:

$$50 - Q = 20 + 2Q$$

Solving for Q, we get:

$$3Q = 30 \Rightarrow Q = 10$$

The deadweight loss is the area of the triangle formed by the equilibrium quantity (13.33), the socially optimal quantity (10), and the price difference between the demand and MSC curves. The price change is 36.67 - 30 = 6.67 and the quantity change is 13.33 - 10 = 3.33.

Deadweight Loss Calculation

The formula for deadweight loss is:

$$DWL = rac{1}{2} imes (ext{Price Change}) imes (ext{Quantity Change})$$
 $DWL = rac{1}{2} imes 6.67 imes 3.33 = 11.11$

Therefore, the deadweight loss due to the negative externality is \$11.11.

Policies to Fix Externalities

- Negative externality: standards, taxes
- Positive externality: subsidies, standards
- Why might bans or mandates be a bad way to fix a negative externality?



Figure 7: A tax is a per-unit punch in the face.

Example Setup — Externalities of Vaccines

- **Cost of Sickness**, C_s: The financial and social cost incurred if a person gets sick.
- **Proportion of Population that Gets Sick**, *P_s*: Fraction of the population that contracts the illness.
- **Probability of Sickness for Unvaccinated**, *p_u*: Higher likelihood of sickness for those who do not get vaccinated.
- Probability of Sickness for Vaccinated, p_v: Reduced likelihood of sickness for vaccinated individuals, where p_v < p_u.
- **Cost of Vaccine**, C_v : The monetary and time cost associated with getting vaccinated.

Key Question

Markets Mathematically

What is the efficient level of vaccination when considering individual choices versus social welfare?

Worked Solution — Analyzing Vaccine Externalities

• Individual Cost-Benefit Analysis: An individual will choose to vaccinate if the cost of sickness, weighted by probability, exceeds the vaccine cost.

$$p_u \cdot C_s > C_v$$

• Social Cost-Benefit Analysis: Socially optimal vaccination considers reduced sickness in the population, even among the unvaccinated, due to herd immunity.

Expected Social Cost = $(1 - P_s) \cdot C_v + P_s \cdot (p_v \cdot C_s + (1 - p_v) \cdot C_v)$

- Externality Effect: Each vaccinated person reduces the overall proportion *P_s* of sick individuals, lowering the social cost.
- **Optimal Vaccination Rate:** Set the marginal social benefit of additional vaccination (in reducing $P_s \cdot C_s$) equal to the marginal cost of vaccination, yielding the efficient vaccination rate.

Example Setup — Vaccine Externalities (Numerical)

- Cost of Sickness, C_s: Assume the cost of getting sick is \$500.
- **Probability of Sickness for Unvaccinated**, *p_u*: The probability an unvaccinated person gets sick is 0.2.
- **Probability of Sickness for Vaccinated**, *p_v*: The probability a vaccinated person gets sick is 0.05.
- Cost of Vaccine, C_v : Assume the cost of getting the vaccine is \$50.
- Marginal External Benefit (MEB): Each vaccinated person reduces the probability of sickness for others, valued at an external benefit of \$30 per vaccination.

Key Question

Markets Mathematically

What are the private and socially optimal vaccination rates, and what subsidy could correct the externality?

Worked Solution — Solving the Vaccine Externality (Numerical)

• Individual Decision (Private Optimum): An individual will choose to vaccinate if the private expected cost of sickness exceeds the vaccine cost:

 $p_u \cdot C_s > C_v$

 $0.2\cdot 500 > 50 \Rightarrow 100 > 50$

Since the inequality holds, the individual will choose to vaccinate.

• **Social Optimum:** The socially optimal decision considers the marginal external benefit (MEB) as well:

 $p_u \cdot C_s > C_v - MEB$

 $0.2\cdot 500 > 50-30 \Rightarrow 100 > 20$

Vaccination is more beneficial with the external benefit included, justifying a higher vaccination rate than without it

Internalizing the Externality

 Corrective Subsidy: To internalize the external benefit, offer a subsidy equal to the marginal external benefit (MEB) of \$30:

Optimal Subsidy = MEB = 30

• **Result:** The subsidy effectively lowers the cost of vaccination to \$20, aligning private incentives with the socially optimal level.

Example Setup — Pollution as an Externality

- Marginal Private Cost (MPC): The cost incurred by a firm for producing one additional unit, excluding externalities.
- Marginal External Cost (MEC): The cost of pollution imposed on society by the production of one additional unit, such as health impacts or environmental degradation.
- Marginal Social Cost (MSC): The total cost to society of producing one additional unit, given by:

MSC = MPC + MEC

• Marginal Benefit (MB): The benefit to society of consuming one additional unit.

Key Question

Markets Mathematically

How does the presence of pollution as an externality affect the socially optimal level of production?

Extenalities

Worked Solution — Solving the Pollution Externality

• Socially Optimal Production Level: The efficient quantity, Q^{*}, occurs where Marginal Social Cost (MSC) equals Marginal Benefit (MB):

MSC = MB

• **Private Market Outcome:** Without accounting for external costs, the firm produces where Marginal Private Cost (MPC) equals Marginal Benefit (MB), leading to a higher quantity, Q_p , than the social optimum.

$$MPC = MB$$

• **Deadweight Loss (DWL):** The area between Q_p and Q^* , representing the social cost of overproduction due to the externality.

Policy Solution

 Policy Solution: A tax equal to the Marginal External Cost (MEC) aligns private costs with social costs, shifting MPC to match MSC:

Optimal Tax = MEC

Conclusion

By internalizing the external cost (e.g., with a tax or regulation), the socially optimal production level Q^* can be achieved, reducing the deadweight loss from overproduction.

Example Setup — Pollution as an Externality (Numerical)

• Marginal Private Cost (MPC): Assume the marginal private cost (MPC) of production is given by:

MPC = 10 + 2Q

• Marginal External Cost (MEC): The external cost per unit of pollution (e.g., health and environmental damages) is constant at:

MEC = 5

• Marginal Social Cost (MSC): The total cost to society of producing an additional unit is:

MSC = MPC + MEC = (10 + 2Q) + 5 = 15 + 2Q

• Marginal Benefit (MB): Assume the marginal benefit to society from consumption is given by:

$$MB = 50 - Q \quad \textcircled{P} \quad \textcircled{P}$$

Markets Mathematically

Worked Solution — Solving the Pollution Externality (Numerical)

• Private Market Outcome (No Tax): Firms produce where Marginal Private Cost (MPC) equals Marginal Benefit (MB):

10 + 2Q = 50 - Q

Solving for *Q*:

Markets Mathematically

$$3Q = 40 \Rightarrow Q_p = 13.33$$

• Socially Optimal Outcome: The social optimum occurs where Marginal Social Cost (MSC) equals Marginal Benefit (MB):

$$15 + 2Q = 50 - Q$$

Solving for *Q*:

$$3Q = 35 \Rightarrow Q^* = 11.67$$

Corrective Tax

• **Corrective Tax:** To internalize the externality, impose a tax equal to the Marginal External Cost (MEC):

Optimal Tax = MEC = 5

• **Result:** The tax reduces production from $Q_p = 13.33$ to $Q^* = 11.67$, achieving the socially optimal level.

Conclusion

With the corrective tax, firms face the true social cost, reducing overproduction and aligning with the socially efficient outcome.

Example Setup — Congestion Games

- **Definition:** A congestion game is a type of game where multiple players share a set of resources, and each player's payoff depends on the level of congestion (number of players) on each resource.
- Example Scenario: Commuters choose between two routes, Route A and Route B, to reach their destination. The time it takes depends on how many commuters choose each route.
- Cost Functions:
 - **Route A Cost**, C_A : $C_A = 10 + 2x_A$, where x_A is the number of commuters choosing Route A.
 - Route B Cost, C_B : $C_B = 15 + x_B$, where x_B is the number of commuters choosing Route B.
- **Objective:** Each commuter wants to minimize their travel time by choosing the less congested route.

Worked Solution — Solving the Congestion Game

• Nash Equilibrium Condition: In equilibrium, the travel times on Route A and Route B should be equal; otherwise, commuters would switch to the route with the shorter time.

$$C_A = C_B$$

• Equilibrium Calculation:

- Let the total number of commuters be N = 10.
- Suppose x_A commuters choose Route A, and $x_B = N x_A$ commuters choose Route B.
- Set the costs equal to each other:

$$10 + 2x_A = 15 + (N - x_A)$$

Markets Mathematically

Solving Congestion Game

Solving for x_A:

$$3x_A = 5 + N \Rightarrow x_A = \frac{5 + N}{3}$$

With N = 10, we find $x_A = 5$ and $x_B = 5$.

• **Result:** At Nash equilibrium, 5 commuters choose Route A, and 5 choose Route B. Both routes have the same cost, so no commuter has an incentive to switch.

Conclusion

In congestion games, equilibrium is reached when costs balance across resources, leading to no incentive for players to deviate.

Effects of Superhighway



Figure 8: Are more roads the solution to congestion?

Public Goods

Conclusion

Congestion Game Graphical Example



Figure 9: Basic Congestion Game

What are Public Goods?

- Nonexcludability: People who do not pay for the good cannot be stopped from using it
- Nonrivalry: The use of the good by one person does not exclude use by others

Examples of Public Good

- Public Education
- Basic research
- Clean air and water
- National defense

The Free-Rider Problem

• The market will not provision public goods. Why?



The Free-Rider Problem

- The market will not provision public goods. Why?
- If you cannot exclude someone, you cannot charge for it
- Some people will use good and not contribute relying on others
- Think about unions

The Tragedy of the Commons

- **Concept:** The Tragedy of the Commons occurs when individuals, acting in their own self-interest, overuse a shared resource, leading to its depletion or degradation.
- **Classic Example:** Consider a common pasture shared by multiple herders. Each herder has an incentive to graze as many animals as possible on the pasture to maximize their own benefit.
- **Problem:** As each herder adds more animals, the pasture becomes overgrazed, eventually reducing or destroying the resource for everyone.
- **Outcome:** Without regulation or cooperation, the resource is overused, leading to inefficient outcomes for both individuals and society.

Mathematical Formulation of the Tragedy of the Commons

- Setup: Let *N* herders each decide how many animals *q_i* to graze on a common pasture, where *i* = 1, 2, ..., *N*.
- Total Grazing Intensity: The total number of animals grazing is:

$$Q = \sum_{i=1}^{N} q_i$$

- Benefit per Animal: Each herder receives a benefit B(Q) per animal, where B(Q) decreases as Q increases (reflecting resource depletion).
- Individual Payoff: The payoff to each herder *i* is:

$$\pi_i = q_i \cdot B(Q) - C \cdot q_i$$

where C is the cost of grazing per animal.

Markets Mathematically

Equilibrium

- Nash Equilibrium: Each herder maximizes π_i with respect to q_i, ignoring the impact on B(Q) for others, leading to overgrazing as Q exceeds the socially optimal level.
- **Social Optimum:** The socially optimal level of *Q* occurs where total benefits minus total costs are maximized:

$$\max_{Q} \left(Q \cdot B(Q) - C \cdot Q \right)$$

• **Result:** At Nash equilibrium, *Q* is higher than the social optimum due to individual incentives to overuse the resource.

Conclusion

The Tragedy of the Commons leads to overuse of the resource, demonstrating the need for cooperative management or regulation.

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Mathematical Example of the Free-Rider Problem in Unions

- Setup: A union negotiates improved wages and benefits that benefit all workers in a firm, whether or not they are union members.
- Union Cost and Benefit: Each worker gains a benefit *B* from the union's collective bargaining, but union members pay a membership fee *C*.
- **Payoff for Union Members:** If a worker joins the union, their net payoff is:

 $\pi_{\text{member}} = B - C$

• Payoff for Non-Members (Free Riders): Non-members also receive the benefits from the union's efforts without paying the fee:

 $\pi_{non-member} = B$

Incentive to Free Ride

- Incentive to Free-Ride: Since non-members avoid the cost *C* but still gain *B*, there is an incentive for individuals to free-ride by not joining the union.
- Equilibrium Outcome: If enough workers free-ride, the union may lack resources to negotiate effectively, reducing benefits *B* for all workers.
- Socially Optimal Outcome: The optimal outcome occurs when all workers join and contribute, ensuring strong bargaining power and maximizing *B* through collective action.

Conclusion

The free-rider problem weakens union effectiveness, illustrating the challenge of providing public goods like collective bargaining without incentives for individual contribution.

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Free-rider Problem Climate Change

- If the climate dies, everyone is hurt.
- If the climate is saved, everyone is helped.
- This incentivizes countries to lean on other countries to make investments. Why do your part if you'll benefit regardless?
- Very similar to prisoner's dilemma

Policies for Public Goods

- Public support and funding. National parks, national defense, Research and Development
- Agreements for provision among individuals or countries. Paris Climate accords.
Thank You So Much!

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Conclusion ○●

List of References

